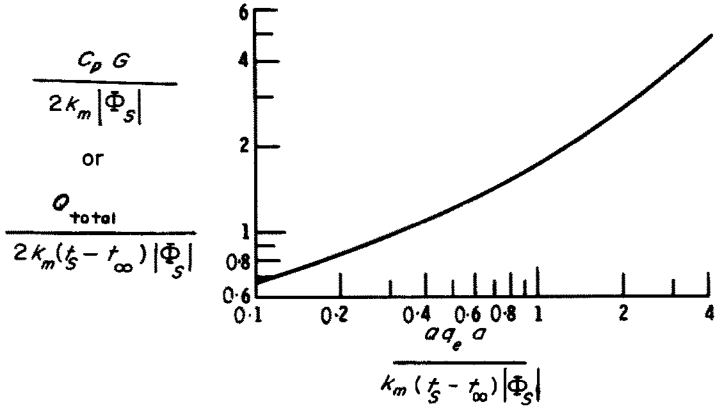

FIG. 5. Dimensionless coolant or heat flow as a function of physical parameter involving absorbed incident radiation.

region having a free boundary can then be utilized to obtain results for a related porous cooled problem. In this way a porous region shape is obtained that will provide proper cooling for a specified heat flux variation along a surface while maintaining a specified uniform surface temperature. A two-dimensional example is given where a surface is subjected to thermal radiation from one direction. The analysis applies for three dimensions, but the example is limited to two dimensions because conformal mapping is used to obtain the free boundary shape.

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A GENERAL EXPRESSION FOR THE RATE OF EVAPORATION OF A LAYER OF LIQUID ON A SOLID BODY

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NOMENCLATURE

C, solution of $C\,e^{C^2}\,\text{erfc}\,C = 1/(\lambda\sqrt{\pi})$;
C_1, constant in $R = C_1 t^n$;
C_2, constant in $\delta_0 = C_2\sqrt{(\nu t_g)}$;
c_p, specific heat capacity;
$F(\lambda, \phi, \beta^2, \gamma, t^+)$, function defined in text as $(1 - \delta^+)$;
h_{fg}, latent heat of vaporization;
h_i, surface heat transfer coefficient at interface between liquid and vapour;

$I(\lambda, \phi, \beta^2, \gamma, m, n)$,	integral defined in text;
J,	Jakob number $(\rho_l/\rho_v)(c_{pl}(T - T_{sat})/h_{fg})$; Suffix $_w$ or $_b$ applies to T;
k,	thermal conductivity;
m	$C_2^2 Pr$;
n,	exponent in $R = C_1 t^n$;
Pr,	Prandtl number (ν/α);
$(\dot{q}/A)$,	heat flux;
R,	radius of bubble;
t,	time;
T,	temperature;
V,	volume;
x,	length coordinate (normal distance from liquid–solid interface);
α,	thermal diffusivity $(k/\rho c_p)$ of liquid;
β,	$\left\{\frac{k_s\rho_s c_{ps}}{k_l\rho_l c_{pl}}\right\}^{\frac{1}{2}}$;
γ,	$k_l/(h_i\delta_0)$;
δ,	thickness of microlayer;
λ,	$\frac{h_{fg}}{c_p(T_{w0} - T_{sat})}$;
ϕ,	$\frac{\delta_0(\dot{q}/A)_0}{k_l(T_{w0} - T_{sat})}$;
ν,	kinematic viscosity;
ρ,	density.

Suffices

b,	bulk liquid;
g,	growth;
i,	interface, liquid–vapour;
l,	liquid;
m,	microlayer;
0,	initial;
s,	solid;
sat,	saturation;
v,	vapour;
w,	wall surface.

INTRODUCTION

It has been shown [1] that when bubbles of vapour form during boiling of a liquid at a heated flat wall, the rate of growth of the bubbles is in some cases influenced by evaporation from a thin layer of liquid (the microlayer) which forms beneath the bubble. Early calculations of that evaporation [2–8] have all assumed the microlayer to be initially stationary and also used two or more of the following simplifications when determining the rate of flow of heat from the wall through the microlayer:

neglecting the thermal capacity of the microlayer
neglecting variation of temperature in the wall
neglecting initial heat flux in the system
assuming the microlayer to be "infinitely thick"
neglecting the temperature drop at the liquid–vapour interface which arises from the finite rate of evaporation

A numerical calculation has been carried out [9] which determines the rate of evaporation from an initially stationary microlayer, avoiding these simplifications, and the results are presented here. Wide ranges of possible combinations of properties of wall and fluid are covered by considering appropriate ranges of dimensionless groups.

The aim was to provide a basis from which calculations of bubble growth can be made, by methods such as [6, 7], without any need to simplify this part of the analysis. For such purposes it is preferable that the results should be an analytic expression, rather than a graphical or tabulated computer output. In a few cases, which are extreme or are special in some other way, there are exact analytic solutions. By taking these as a basis, it has proved possible to set up a single analytic expression which represents all cases within a wide range, with an error (± 15 per cent) which is generally less than the other errors involved in incorporating this element into a theory of bubble growth

FORMULATION OF ONE-DIMENSIONAL MODEL

The equations and initial and boundary conditions governing the one-dimensional flow of heat through a solid and evaporating liquid as developed in [6] and [9], are presented in the appendix in non-dimensional form. At least four non-dimensional groups are involved, and the following are convenient:

$$\beta^2 = \frac{k_s\rho_s c_{ps}}{k_l\rho_l c_{pl}}$$ the ratio of thermal properties of solid and liquid

$$\phi = \frac{(\dot{q}/A)_0\delta_0}{k_l(T_{w0} - T_{sat})}$$ the dimensionless wall heat flux

$$\lambda = \frac{h_{fg}}{c_{pl}(T_{w0} - T_{sat})}$$ dimensionless latent heat

$$\gamma = \frac{k_l}{h_i\delta_0}$$ dimensionless resistance to heat flow at the liquid–vapour interface.

The equations have been solved numerically for a wide range of these four groups, and the results are presented dimensionlessly in terms of reduction in microlayer thickness $(1 - \delta^+)$ where $\delta^+ = \delta/\delta_0$ against time t^+ $(= \alpha t/\delta_0^2)$. These results are taken to define a function $F(\lambda, \phi, \beta^2, \gamma, t^+) = (1 - \delta^+)$, and three typical sets of graphs are presented here in Fig. 1. More detailed results were presented in [6], for $\lambda = 5$, 50, $\phi = 0{\cdot}03$, $0{\cdot}3$, $\beta^2 = 1$, 10, 1000 and $\gamma = 0$. In [8] the range was extended to include $\beta^2 = 0$ and ∞. In [6] and [8] computed values of wall temperature (T_w^+) as a function of t^+ were also presented, and these compare favourably with experimentally measured wall temperature transients reported in [10]. In [9] an intermediate value of 15 was included for λ, the range of ϕ was extended to 1·0 and the effect of γ was included, taking values 0·03 and 0·3. Selection of these values for γ involved the current uncertainty

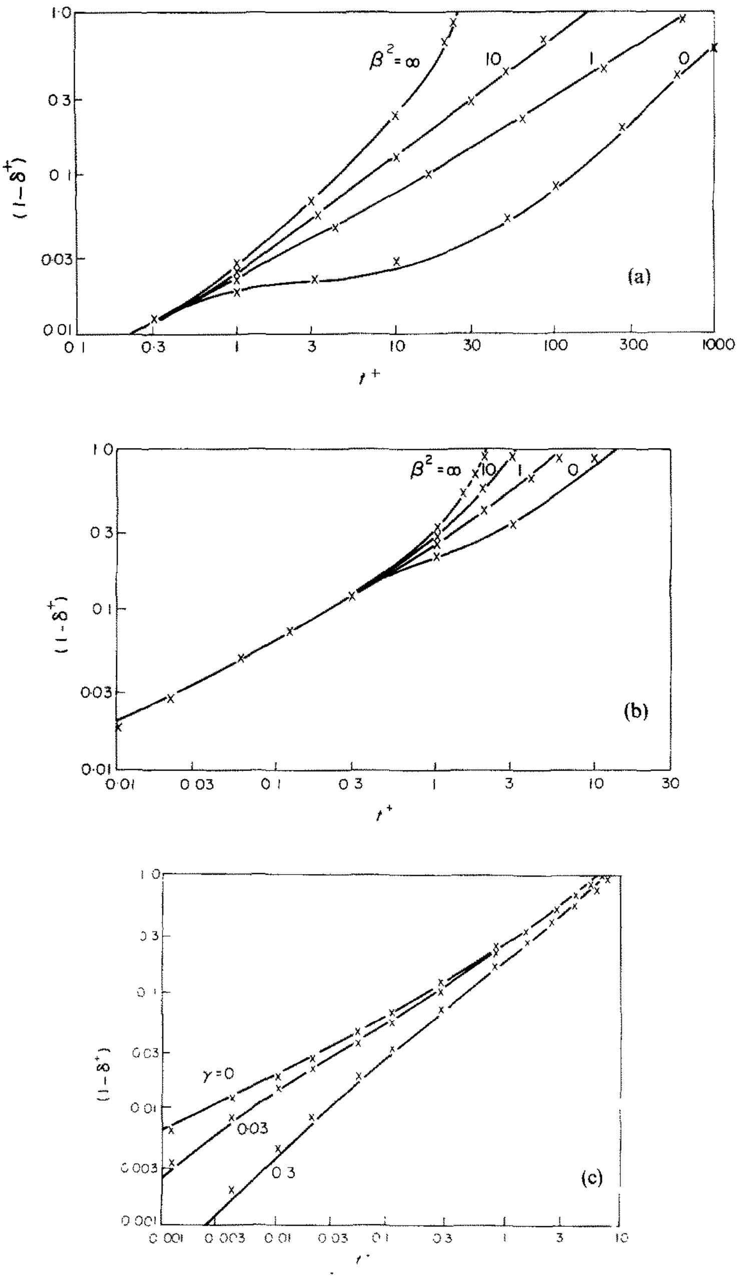

FIG. 1. Dimensionless reduction of microlayer thickness $(1 - \delta^+)$ against dimensionless time (t^+). Full lines represent the numerical results, crosses represent the approximate expression. (a) $\phi = 0{\cdot}03$, $\lambda = 50$, $\gamma = 0$.
(b) $\phi = 0{\cdot}3$, $\lambda = 5$, $\gamma = 0$.
(c) $\phi = 0{\cdot}3$, $\lambda = 5$, $\beta^2 = 1$.

concerning the value of h_i and the associated accommodation coefficient. As discussed in [9], recent evidence from [11] suggests that, for water, γ is less than 0·1 for pressures exceeding 0·1 atm and for bubble life exceeding 0·1 ms.

ANALYTIC SOLUTIONS AND APPROXIMATIONS

Exact analytic solutions are obtainable in various simple or extreme cases. Three such cases have been considered in [9] and the solutions combined to give an approximate analytical expression for F. The three cases are:

(i) Allowing for movement of the interface (i.e. λ unrestricted) but taking $\phi = 0$, $\beta^2 = 1$ and $\gamma = 0$. The solution can be derived from [12] §11.2, and is

$$F = (1 - \delta^+) = 2C\sqrt{(t^+)} \simeq \frac{1}{\lambda - 2/\pi}\frac{2}{\sqrt{\pi}}\sqrt{(t^+)}$$

where C is the solution of $Ce^{C^2}\,\mathrm{erfc}\,C = 1/(\lambda\sqrt{\pi})$ and the approximation is within 1 per cent for $\lambda > 4$.

(ii) Allowing for variation of ϕ and β^2 but neglecting movement of the interface (i.e. λ large) and taking $\gamma = 0$. The problem reduces to conduction through a composite slab, for which a solution is given in [12] § 12.8, yielding

$$F = (1 - \delta^+) = \frac{\phi}{\lambda}t^+ + \frac{2}{\sqrt{(\pi)}}\cdot\frac{1-\phi}{\lambda}\sqrt{(t^+)} \times \left[1 + 2(\sqrt{\pi})\sum_{s=1}^{\infty}\left(\frac{\beta - 1}{\beta + 1}\right)^s \mathrm{ierfc}\,\frac{s}{\sqrt{(t^+)}}\right].$$

(iii) Allowing for variation of ϕ and γ but again neglecting movement of the interface and also taking $\beta^2 = 1$. This problem reduces to conduction through a single body, for which a solution is given in [12] § 2.7, yielding

$$F = (1 - \delta^+) = \frac{\phi}{\lambda}t^+ + \frac{1 - \phi(1+\gamma)}{\lambda}\left\{\frac{2}{\sqrt{(\pi)}}\sqrt{(t^+)} - \gamma(1 - e^{t^+/\gamma^2}\,\mathrm{erfc}\frac{\sqrt{(t^+)}}{\gamma}\right\}.$$

In all three cases the expression for F is valid only if it is less than 1. If the expression exceeds 1, then F is taken to be 1.

When these three analytic expressions and the numerical results are considered together, some suggestions arise:

(a) the effect of the movement of the interface might be largely allowed for by writing $(\lambda - 2/\pi)$ instead of λ.

(b) since γ has most effect when $t^+ < \gamma^2 \ll 1$, and β^2 has most effect when $t^+ \simeq 1$ or greater, the effects of γ and β^2 (together with ϕ) might be largely allowed for by an expression combining the square brackets of (ii) and the curved brackets of (iii). Thus the suggested approximate expression is:

$$F = (1 - \delta^+) = \frac{\phi}{\lambda - 2/\pi}t^+ + \left(\frac{1 - \phi(1+\gamma)}{\lambda - 2/\pi}\right.$$
$$\times\left[1 + 2(\sqrt{\pi})\sum_{s=1}^{\infty}\left(\frac{\beta-1}{\beta+1}\right)^s \mathrm{ierfc}\,\frac{s}{\sqrt{(t^+)}}\right]$$
$$\left.\times\left\{\frac{2}{\sqrt{(\pi)}}\sqrt{(t^+)} - \gamma(1 - e^{t^+/\gamma^2}\,\mathrm{erfc}\frac{\sqrt{(t^+)}}{\gamma}\right\}\right).$$

Agreement between this expression and the numerical solution is good provided $0 < \beta^2 < 1$, and also good for larger values of β^2 until F exceeds about 0·3. A reasonably simple modification which improves agreement for $F > 0·3$ with $\beta^2 > 1$ is to multiply the expression above by

$$\left\{1 + \frac{\beta - 1}{\beta + \lambda}\left(\frac{2t^+}{\lambda}\right)^2\right\}\ \text{when}\ \beta^2 > 1$$

The resulting expression has been calculated for many cases throughout the range; results are shown as crosses on the graphs of Fig. 1. As shown in these graphs and by more detailed consideration in [9], the approximation is generally within a few per cent, and always within ±15 per cent throughout the range $5 < \lambda < 50$, $0 < \phi < 1$, $0 < \gamma < 0·3$, all β^2.

APPLICATION TO BUBBLE GROWTH RATE

In [6] a general expression is derived for the volume (V_m) of vapour evaporated from the microlayer of a hemispherical bubble, assuming that its radius grows as a constant power of time ($R = C_1t^n$) and assuming also (after [3]) that $\delta_0 = C_2\sqrt{(\nu t_g)}$. The expression is:

$$V_m = 2\pi R^{(2+\frac{1}{2}n)}\frac{\rho_l}{\rho_v}I\frac{nC_2\sqrt{(\nu)}}{C_1^{(\frac{1}{2}n)}}$$

where

$$I(\lambda, \phi, \beta^2, \gamma, m, n) = \int_0^{\infty}\frac{F(\lambda, \phi, \beta^2, \gamma, t'/m)}{(1 + t')^{\frac{3}{2}+2n}}\,dt'$$

where $m = C_2^2 Pr$ and a transformation $t' = (t/t_g - 1)$ has been made.

Evaluation of I is in general complicated by the variation of δ_0 with radius, causing ϕ and γ to depend on radius and hence on t'. These complications can be avoided by assuming that the effects of ϕ and γ are negligible (i.e. $\phi = \gamma = 0$). If we also assume that evaporation of the microlayer is the sole source of vapour for growth of the bubble, then $V_m = \frac{2}{3}\pi R^3$ hence $n = \frac{1}{2}$ and C_1 is determined, giving an expression for bubble growth:

$$R = \tfrac{3}{2}C_2\sqrt{(\nu t)}\frac{\rho_l}{\rho_v}I = \tfrac{3}{2}J_w\sqrt{(\alpha t)}\{\lambda I\sqrt{(m)}\}.$$

For these cases, with $\phi = \gamma = 0$, $n = \frac{1}{2}$, the integral I was evaluated for some values of λ, β^2, m in [6] and more generally in [9], using the analytic expression derived above for F. Results are given in Table 1 below, in terms of the group $\{\lambda I\sqrt{(m)}\}$. As shown in [3], p. 929, the effect of evaporation from the curved surface of the bubble is readily included, provided the liquid was initially at uniform temperature T_b. The effect is to add a second term $2\sqrt{(3/\pi)}J_b\sqrt{(\alpha t)}$ to the radius.

Table 1. *Values of* $\{\lambda\sqrt{(m)}\,I(\lambda, \phi, \beta^2, \gamma, m, n)\}$ *with* $\phi = \gamma = 0$ *and* $n = \frac{1}{2}$

	$\lambda = 5$			$\lambda = 50$		
m \ β^2	1	10	∞	1	10	∞
4	0·85	0·97	1·07	0·76	0·85	1·00
1	0·83	1·07	1·13	0·76	1·01	1·48
$\frac{1}{2}$	0·81	1·09	1·24	0·76	1·14	1·90

Analyses of this type fail when the effects of ϕ and γ are taken into account, as it will no longer be valid to take $R = C_1 t^n$. It appears that a less analytic approach will be needed, and a step-by-step numerical technique is probably required, for which the analytic expression given above for F may be of value.

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APPENDIX

Formulation in Non-dimensional Terms

Taking as non-dimensional variables:

$$T^+ = \frac{T - T_{sat}}{T_{w0} - T_{sat}} \qquad t^+ = t\frac{\alpha_1}{\delta_0^2}$$

$$x_l^+ = \frac{x_1}{\delta_0} \qquad x_s^+ = \frac{x_s k_1}{\delta_0 k_s} \qquad \delta^+ = \frac{\delta}{\delta_0}$$

the equations, initial conditions and boundary conditions become:

$$\left.\begin{array}{lll}
\dfrac{\partial^2 T_l^+}{\partial x_l^{+2}} = \dfrac{\partial T_l^+}{\partial t^+} & \text{in liquid} \\[2ex]
\dfrac{\partial^2 T_s^+}{\partial x_s^{+2}} = \beta^2 \dfrac{\partial T_s^+}{\partial t^+} & \text{in solid} \\[2ex]
\left.\begin{array}{l} T_l^+ = T_s^+ \\ \dfrac{\partial T_l^+}{\partial x_l^+} = -\dfrac{\partial T_s^+}{\partial x_s^+} \end{array}\right\} & \text{at } x_l^+ = x_s^+ = 0 \\[2ex]
\dfrac{\partial T_s^+}{\partial x_s^+} = \phi & \text{at large } x_s^+
\end{array}\right\} \text{for } t^+ > 0$$

$$\left.\begin{array}{ll}
T_l^+ = 1 - \phi x_l^+ & 0 < x_l^+ < \delta^+ \\
T_s^+ = 1 + \phi x_s^+ & \\
\delta^+ = 1 &
\end{array}\right\} \text{at } t^+ = 0$$

$$\frac{\partial T_l^+}{\partial x_l^+} = \lambda\frac{\partial \delta}{\partial t^+} = -\frac{T_l^+}{\gamma} \quad \text{at } x_l^+ = \delta^+ \quad \text{for } t^+ > 0$$

where the four non-dimensional groups, β^2, ϕ, λ and γ are defined in the text.